

# RAMAPO-INDIAN HILLS SCHOOL DISTRICT

Dear Ramapo---Indian Hills Student:

Please find attached the summer packet for your upcoming math course. The purpose of the summer packet is to provide you with an opportunity to review prerequisite skills and concepts in preparation for your next year's mathematics course. While you may find some problems in this packet to be easy, you may also find others to be more difficult; therefore, you are not necessarily expected to answer every question correctly. Rather, the expectation is for students to put forth their best effort, and work diligently through each problem.

To that end, you may wish to review notes from prior courses or on-line videos ([www.KhanAcademy.com](http://www.KhanAcademy.com), [www.glencoe.com](http://www.glencoe.com), [www.youtube.com](http://www.youtube.com)) to refresh your memory on how to complete these problems. We recommend you circle any problems that cause you difficulty, and ask your teachers to review the respective questions when you return to school in September. Again, given that math builds on prior concepts, the purpose of this packet is to help prepare you for your upcoming math course by reviewing these prerequisite skills; therefore, the greater effort you put forth on this packet, the greater it will benefit you when you return to school.

Please bring your packet and completed work to the first day of class in September. Teachers will plan to review concepts from the summer packets in class and will also be available to answer questions during their extra help hours after school. Teachers may assess on the material in these summer packets after reviewing with the class.

If there are any questions, please do not hesitate to contact the Math Supervisors at the numbers noted below.

Enjoy your summer!

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## AP CALCULUS AB

### SUMMER PACKET

Dear Future AP Calculus AB student,

First and foremost, welcome to AP Calculus and congratulations on your enrollment, which reflects a testament to your hard work and mathematic achievements over your high school career. This letter and assignment serves to enlighten you on the requirements and expectations for the course, and provides you with an opportunity to review and hone in on the skills necessary for success in calculus.

AP Calculus AB prepares students for the national college board examination administered in May. Therefore, enrollment in the course necessitates the following expectations for the students: they have mastered the prerequisite skills noted below, they maintain an enthusiastic and conscientious attitude and work ethic for the duration of the course, they understand and can handle the pacing and work load the course requires to adequately prepare students for the exam, and lastly, they register and sit for the AP examination in May. The main topics on the exam include differential and integral calculus; therefore, limited time in class is spent reviewing elementary functions; refreshing these skills is done by the student over the summer.

### Prerequisites

Before studying calculus, all students should complete four years of secondary mathematics designed for college-bound students: courses in which they study algebra, geometry, trigonometry, analytic geometry, and elementary functions. These functions include those that are **linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise defined**. In particular, before studying calculus, students must be familiar with the properties of functions, the algebra of functions, and the graphs of functions. Students must also understand the language of functions (domain and range, odd and even, periodic, symmetry, zeros, intercepts, and so on) and know the values of the trigonometric functions of the numbers 0,  $\pi/6$ ,  $\pi/4$ ,  $\pi/3$ ,  $\pi/2$ , and their multiples.

### Course Goals

*Students should be able to:*

- work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal. They should understand the connections among these representations.
- understand the meaning of the derivative in terms of a rate of change and local linear approximation and they should be able to use derivatives to solve a variety of problems.
- understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change and should be able to use integrals to solve a variety of problems.
- understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.
- model a written description of a physical situation with a function, a differential equation, or an integral.
- use technology to help solve problems, experiment, interpret results, and verify conclusions.
- develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

As stated above, most of the year must be devoted to topics in *differential* and *integral* calculus. These topics are the focus of the AP Exam. Therefore, the summer assignments listed below are designed to help you review topics from algebra, geometry, and precalculus so that when you arrive in September, you are ready to review the first main theme in Calculus: *limits and continuity*. All assignments will be collected the first week back of school

DIRECTIONS: Solve each problem below. Write neatly. Show all work. Box in your final answers.

1. Test for Symmetry with respect to each axis and to the origin.

a)  $y = x^2 - 6$

b)  $y = \frac{x}{x^2+1}$

2) Sketch the graph of the equation. Identify any intercepts and test for symmetry

a)  $y = 2 - 3x$

b)  $y = 9 - x^2$

c)  $y = x^3 + 2$

d)  $y = 6 - |x|$

3. Find the points of intersection of the graphs of the equations.

a)  $x + y = 8, 4x - y = 7$

b)  $x^2 + y = 6, x + y = 4$

c)  $y = x^3, y = x$

4) Determine whether the statement is true or false. If it is false, explain why or given an example that shows it is false.

a) If  $(-4, -5)$  is a point on a graph that is symmetric with respect to the x-axis, then  $(4, -5)$  is also a point on the graph.

b) If  $(-4, -5)$  is a point on a graph that is symmetric with respect to the y-axis, then  $(4, -5)$  is also a point on the graph.

c) If the discriminant in a quadratic equation is greater than zero, then the graph of the quadratic has two x-intercepts.

d) If the discriminant in a quadratic equation equals zero, then the graph of the quadratic has only one x-intercept.

5. Find an equation of the line that passes through the point and has the indicated slope.

a)  $(0, 3)$  slope  $= \frac{3}{4}$

b)  $(3, -2)$   $m = 3$

6. Find an equation of the line that passes through the points and sketch the line.

a)  $(2, 8)$  and  $(5, 0)$

b)  $(6, 3)$  and  $(6, 8)$

7. Find an equation of the vertical line with x-intercept at 3

8. Sketch a graph of the equation.

a)  $y = -3$

b)  $y - 2 = \frac{3}{2}(x - 1)$

9. Write the general forms of the equations of the lines through the point a) parallel to the given line and b) perpendicular to the given line.

a) point:  $(-7, -2)$  line:  $x = 1$

b) point:  $(2, 1)$  line:  $4x - 2y = 3$

10. Evaluate (if possible) the function at the given value(s) of the independent variable. Simplify the results.

a) For  $f(x) = 5 - x^2$ , find  $f(0)$ ,  $f(\sqrt{5})$ ,  $f(-2)$ ,  $f(x - 1)$

b) For  $f(x) = x^3$ , find  $\frac{f(x+\Delta x) - f(x)}{\Delta x}$

c) For  $f(x) = \cos 2x$ , find  $f(0)$ ,  $f\left(-\frac{\pi}{4}\right)$ ,  $f\left(\frac{\pi}{3}\right)$

d) For  $f(x) = \frac{1}{\sqrt{x-1}}$ , find  $\frac{f(x) - f(2)}{x-2}$

11. Find the domain of the function

a)  $f(x) = \sqrt{6x}$

b)  $y = \sec \frac{\pi t}{4}$

c)  $f(x) = \frac{3}{x}$

12. Evaluate the function as indicated. Given  $f(x) = \{|x| + 1, x < 1, -x + 1, x \geq 1\}$ , find  $f(-3)$ ,  $f(1)$ ,  $f(3)$ ,  $f(b^2 + 1)$

13. Sketch a graph of the function and state its domain and range.

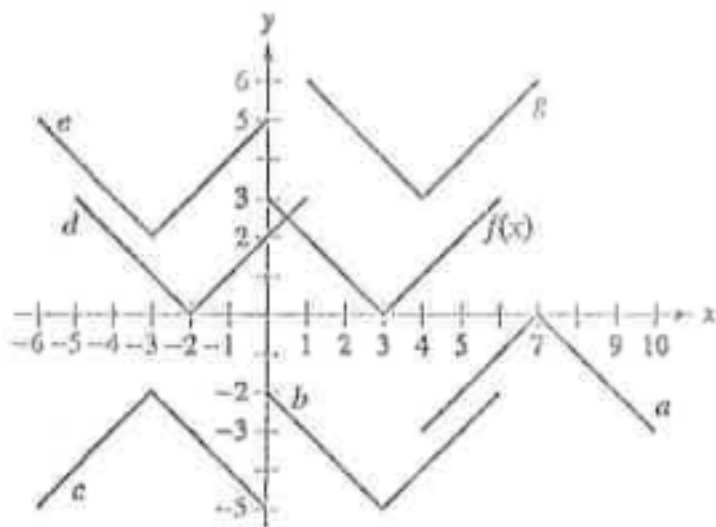
a)  $h(x) = \sqrt{x - 6}$

b)  $f(x) = \sqrt{9 - x^2}$

c)  $g(t) = 3\sin \pi t$

14. Use the graph of  $y = f(x)$  to match the function with its graph:

$$y = f(x + 5), \quad y = f(x) - 5, \quad y = -f(-x) - 2, \quad y = -f(x - 4), \quad y = f(x + 6) + 2, \quad y = f(x - 1) + 3$$



15. Given  $f(x) = \sqrt{x}$  and  $g(x) = x^2 - 1$ , find each:

a)  $f(g(1))$

b)  $g(f(1))$

c)  $g(f(x))$

16. Given  $f(x) = \sin x$  and  $g(x) = \pi x$ , evaluate each expression:

a)  $f(g(2))$

b)  $g(f(0))$

c)  $f(g(x))$



17. Factor and simplify. Express the answer as a fraction without negative exponents.

$$3x(2x + 5)^{-\frac{1}{2}} + 3(2x + 5)^{\frac{1}{2}}$$

18. Express as a simpler fraction

$$\frac{\frac{3}{2(x+h)} - \frac{3}{2x}}{h}$$

19. Multiply

$$(x^{\frac{5}{2}} + \frac{3}{\sqrt{2}})^2$$

20. Solve for  $p$

$$hp - 1 = q + kp + 6p$$

21. Solve for  $x$

$$3(x + 2)^{-1} - \frac{4}{x} = 0$$

22. Solve for  $x$

a)  $\ln(e^{-x}) = 3$

b)  $\frac{e^{2x+3}}{e^3} = 5$

c)  $(e^5)^{3x} = e^5 e^{3x}$

d)  $4^{2x} - 2 \cdot 4^{(x+4)} + 4^8 = 0$

23. Use the properties of logarithms to expand the expression

$$\ln \frac{(3x^2 + 2)\sqrt{x+8}}{(x-1)^4}$$

24. Solve for  $x$

$$\ln \ln x - \ln \ln(x-1) = 1$$

25. Find all  $\theta$  in the interval  $[0, 2\pi]$  that satisfy the equations:  $\sin 2\theta = \frac{1}{2}$

26. Simplify  $\frac{\sec \sec \theta}{\tan \tan \theta}$

27. Answer the following questions about the indicated functions.

Function	Domain	Range	Zeros	Symmetry with respect to y-axis or origin	Even or Odd Functions
a) $f(x) = x^2$					
b) $f(x) = x^3$					
c) $f(x) =  x $					
d) $f(x) = \sin x$					
e) $f(x) = \cos x$					
f) $f(x) = \tan x$					
g) $g(x) = \sec x$					
h) $f(x) = 2^x$					
i) $f(x) = \frac{1}{x}$					
j) $f(x) = \sqrt{x}$					